Linear temperature dependence of the conductivity in Si two-dimensional electrons near the apparent metal-to-insulator transition

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In a high mobility two-dimensional electron system in Si, near the critical density, $n_c=0.32\times10^{11}$ cm^{-2}, of the apparent metal-to-insulator transition, the conductivity displays a linear temperature (T) dependence around the Fermi temperature. When σ_0, the extrapolated $T=0$ conductivity from the linear T dependence, is plotted as a function of density, two regimes with different $\sigma_0(n)$ relations are seen, suggestive of two different phases. Interestingly, a sharp transition between these two regimes coincides with n_c, and σ_0 of the transition is $\sim e^2/h$, the quantum conductance, per square. Toward $T=0$, the data deviate from linear $\sigma(T)$ relation and we discuss the possible percolation type of transition in our Si sample.

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The nature of the ground state of an interacting two-dimensional electron system (2DES) in the presence of disorder is a long-standing problem in condensed matter physics. More than a decade ago, an apparent 2D metal-to-insulator transition (MIT) was first reported as the density of a Si metal-oxide-semiconductor field-effect transistor (MOSFET) is reduced through a characteristic density n_c.[1] Despite much research effort later in the field,[2,3] there remain several unsettled fundamental questions. For example, metallic behavior is observed for density $n>n_c$. Does this metallic-like state persist down to $T=0$ and thus represent a true 2D metal? Furthermore, is this phenomenon related to a phase transition due to strong electron-electron (e-e) interactions,[3] or is it a mundane crossover just due to a complex combination of many well-understood physical mechanisms?[4]

Most of previous work was focused on the e-e interaction effect on metallic side of MIT.[2–6] In recent years, however, it is becoming clear that in the regime close to the Wigner crystallization the strong e-e interaction also plays an important role in electronic transport behavior.[7,8] In ultraclean 2D hole systems realized in the heterostructure insulating-gate field-effect transistors (HIGFETs),[7,8] at the carrier densities around $\sim10^9$ cm^{-2}, the 2D conductivity displayed a roughly linear T dependence near the Fermi energy. This linear dependence does not fit into any known single-particle picture and thus highlights the role of strong e-e interactions.

So far, almost all the experiments in this ultralow density limit have been exclusively carried out in the GaAs-based system, where the high sample quality allows one to approach an electron density down to 0.7×10^9 cm^{-2}.[8] On the other hand, compared to many experiments already carried out in the conventional Si-MOSFETs, the studies in the ultralow density regime have been virtually zero, mainly due to their relatively poor sample quality [in Si-MOSFETs, the highest mobility hovers around 4×10^4 cm^2/Vs, while in GaAs HIGFETs the 2D hole mobility was reported to reach 1.8×10^6 cm^2/Vs (Ref. 7)]. With the advent of high-quality Si/SiGe heterostructures, it finally becomes possible to probe electron transport in low-density regime in the Si-based system, where the electron system is close to the regime of Wigner crystallization with strong e-e interactions.

In an earlier publication, we reported the observation of an apparent 2D MIT and the high-density metallic-like state and its response to an in-plane magnetic field in a high mobility Si quantum well.[9] In this Brief Report, we focus on the transport properties for densities close to the transition. Here, the conductivity displays a linear temperature dependence near $T=T_F$ (the Fermi temperature) and the slope is the same for all different densities around n_c. When the extrapolated $T=0$ conductivity σ_0 of this T dependence is plotted as a function of density, two regimes with linear σ_0 vs n relations are readily seen. Interestingly, these two $\sigma_0(n)$ lines cross each other exactly at n_c and σ_0 at the crossing coincides with the quantum conductance e^2/h per square. At low temperatures, $T\ll T_F$, the measured $\sigma(T)$ deviates from the linear T dependence. We discuss the low-T behavior of our data within the percolation model.

The experiments were performed on the 2DES in an n-type Si quantum well confined in a $Si_{0.75}Ge_{0.25}/Si/Si_{0.75}Ge_{0.25}$ heterostructure. The 2D electron density is tuned continuously by applying a front gate voltage to our field-effect transistor device. Details of the growth and the sample structure can be found in Ref. 10. Standard low-frequency (~7 Hz) lock-in techniques were used to measure the 2D resistivity ρ. At $T\sim300$ mK and zero gate voltage, the 2DES has a density $n=1.45\times10^{11}$ cm^{-2} and mobility $\mu=190\,000$ cm^2/Vs.

In Fig. 1(a), we reproduce selected T-dependence data $\rho(T)$ from our previous paper.[9] Toward the $T=0$ limit, the apparent 2D MIT is clearly seen at the critical density $n_c=0.32\times10^{11}$ cm^{-2}, where $d\rho/dT\sim0$ for $T<1$ K. The insulating behavior, $d\rho/dT<0$, is observed for densities below n_c and metallic-like behavior, $d\rho/dT>0$, above n_c. For further insights to the 2D MIT, we now focus on the data around n_c and plot the inverse resistivity, or conductivity σ, as a function of temperature in the density range of $0.27\times10^{11}<n<0.38\times10^{11}$ cm^{-2} in Fig. 1(b). In the low-T limit, the MIT is again observed in that $d\sigma/dT$ changes sign as n changes through n_c. At high temperatures when T approaches the Fermi temperature T_F, marked as short lines for each density, however, all $\sigma(T)$ curves show roughly a linear T dependence and bends slightly downward for T sufficiently higher than T_F.

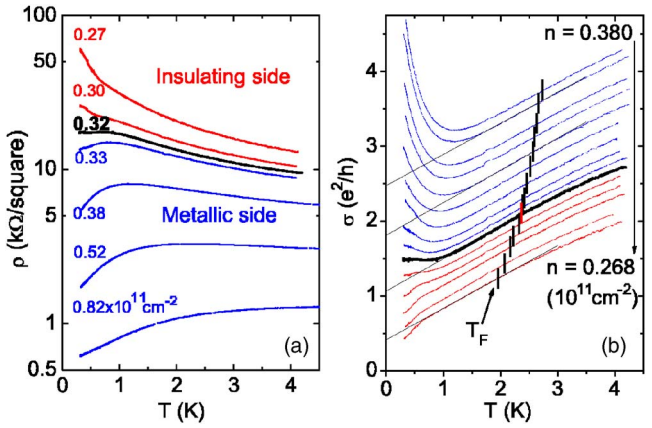

FIG. 1. (Color online) (a) Selected data of $\rho(T)$ in a high-quality Si quantum well reproduced from Fig. 1 in Ref. 5. (b) $\sigma(T)$ in the vicinity of the critical density. The electron densities, from bottom to top, are 0.268, 0.282, 0.296, 0.303, 0.318, 0.322, 0.324, 0.328, 0.336, 0.344, 0.352, 0.358, 0.365, 0.374, and 0.380, in units of 10^{11} cm^{-2}, respectively. The short, vertical lines mark the Fermi energy at each density. The lines on some curves are the linear fits around $T=T_F$.

To illustrate this linear $\sigma(T)$ relation around $T=T_F$,[11] we show the linear fits, $\sigma(T)=\sigma_0+\gamma T$, for several densities, where γ is the slope and σ_0 the linear extrapolation to $T=0$. In the inset of Fig. 2, γ is plotted as a function of the density. It is nearly constant, $\sim 0.43\pm 0.01 e^2/h$ per Kelvin, for $n<0.38\times 10^{11}$ cm^{-2} and then rapidly decreases for higher densities. We plot σ_0 vs n in Fig. 2 and two regimes with different linear $\sigma_0(n)$ relations are readily identified. On the low-density insulating side, σ_0 increases as increasing density at a rate of $11e^2/h$ per 10^{11} cm^{-2}.The slope roughly doubles to $26e^2/h$ per 10^{11} cm^{-2} on the high-density side. Strikingly, the $\sigma_0(n)$ data show a sharp bend at the crossing of the two straight lines at $n=0.32\times 10^{11}$ cm^{-2}, which coincides with n_c of the 2D MIT. And σ_0 at this density exactly equals the quantum conductance e^2/h per square. For both regimes, we extrapolate the linear $\sigma_0(n)$ to zero conductivity at $n_1=0.23\times 10^{11}$ cm^{-2} on the low-density side and $n_2=0.28\times 10^{11}$ cm^{-2} on the high-density side.

Linear $\sigma(T)$ relation has been observed before in p-type GaAs samples.[7,8] There, $\gamma=d\sigma/dT\sim 3$ to $5e^2/h$ per Kelvin, compared to the much smaller number, $\sim 0.43e^2/h$ per Kelvin, in our Si quantum well sample. This one order of magnitude difference might be related to the different Fermi temperatures in these two systems. As will be shown below, $d\sigma/dT\sim 1/T_F$. In our Si sample, T_F ($\sim$2 to 2.5 K) is about a factor of 10 larger than that ($\sim$0.1 to 0.4 K) in the p-GaAs samples. As a result, $d\sigma/dT$ in Si is expected to be $\sim$10 times smaller than in p-GaAs.

Theoretically, the linear $\sigma(T)$ in the high-temperature regime has also been addressed by several models. Das Sarma and Hwang[12] calculated the temperature dependence of σ within a classical model of the screened charged impurity scattering. For $T\gg T_F$, they observed that $\sigma(T)\sim T/T_F\propto T$, consistent with our experimental observation. Besides the screening model, the microemulsion model proposed by Spivak and Kivelson[13] can also explain the linear $\sigma(T)$ at high

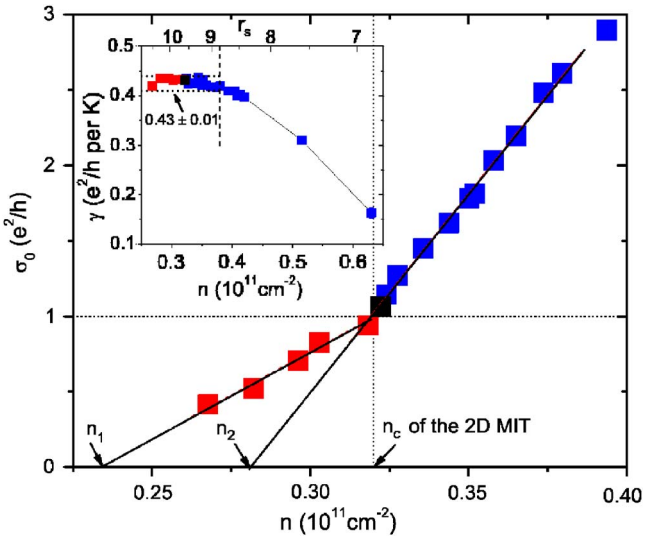

FIG. 2. (Color online) The $T=0$ conductivity σ_0, extrapolated from the high-T linear fit in Fig. 1(b), is plotted as a function of the 2DES density. The solid lines are linear fits on both low-n and high-n regimes and they extrapolate to $\sigma_0=0$ at $n_1=0.23\times 10^{11}$ cm^{-2} and $n_2=0.28\times 10^{11}$ cm^{-2}, respectively. The dotted lines show that the two linear fits cross at n_c and $\sigma_0=e^2/h$. The inset shows the slope γ of the linear $\sigma(T)$ in Fig. 1(b), as a function of density, is roughly constant below 0.38×10^{11} cm^{-2}, and decreases rapidly for higher densities.

temperatures. In this model, near n_c, the ground state of the 2DES consists of an electron liquid with Wigner crystal inclusions on the metallic side, and Wigner solid with liquid inclusions on the insulating side. At high temperatures, Wigner crystal droplets melt and the system behaves classically. In analogy to the physics of ^{3}He near the crystallization pressure,[13] the viscosity of the electron liquid, which is directly proportional to the resistivity, is inversely proportional to T, resulting in a linear $\sigma(T)$, again consistent with our observation.[14] Finally, we notice that a linear T dependence can also be obtained in Ref. 5. Around $T\sim T_F$ and omitting the contribution from $f(T\tau)$ and $t(T\tau)$, one can deduce that $\sigma\propto T$ and the zero-T intercept is just the zero-T Drude conductivity.

Quantitatively, however, none of the three above models can explain the experimental observation that $d\sigma/dT$ is nearly independent of the electron density around n_c. Under the temperature-dependent screening model, $d\sigma/dT$ is proportional to $1/T_F$ or n^{-1}.[12] As a result, the slope should decrease by a factor of $\sim$1.5 when the 2DES density increases from 0.27 to 0.38×10^{11} cm^{-2}. On the other hand, the microemulsion model shows $d\sigma/dT\sim n^2$.[13] Consequently, in the same density range, the slope should change roughly by a factor of 2. Finally, according to Ref. 5, the slope is proportional to τ, which is known to have a strong density dependence in the low-density regime. In contrast, $d\sigma/dT$ is nearly constant and $\sim 0.43\pm 0.01 e^2/h$ per Kelvin in our measurements. So far, it is not known what is responsible for this inconsistency between our experimental result and the theoretical predictions.

Having discussed the linear T dependence of the data, we

need to address some puzzling aspects of the overall results. First, the examined specimen has very high electron mobility and the n_c of the 2D MIT is by far the lowest among all the Si-based samples. The e-e interaction parameter r_s at the transition density is ~10, i.e., the Coulomb energy E_c exceeds the Fermi energy E_F by a factor of 20, after taking into account the twofold valley degeneracy in (001) Si 2DES. Consequently, even though the system behaves classically for $T \sim T_F$, the 2DES is still strongly correlated since E_c is the dominant energy scale here. Second, if the high-T physics underlying the linear $\sigma(T)$ behavior persists to the $T=0$ limit, our $\sigma_0(n)$ data show two distinct regimes, possibly suggesting two different electronic phases below and above n_c. The transition between the two density regimes is sharp, and occurs almost exactly at n_c. σ_0 at this transition point is very close to e^2/h, the unit of quantum conductance, per square, manifesting a possible quantum nature of this transition. The reduction of the rate, $d\sigma_0/dn$, of the density dependence, from $\sim 26e^2/h$ per 10^{11} cm^{-2} above n_c to $\sim 11e^2/h$ per 10^{11} cm^{-2} below n_c, clearly indicates that in the low-density regime the 2D electrons are less likely to become localized by reducing the 2DES density. Considering the large r_s at n_c, one might speculate that strong e-e interactions help to prevent electrons from being localized at low densities. Indeed, in a recent publication,[15] Shi and Xie showed that, with e-e interactions taken into account, the 2DES becomes less localized compared to a noninteracting system at the same density. However, the appearance of two density regimes is still unexplained because a smooth evolution should be expected from their calculations.

We now turn to the low-T limit of the measured $\sigma(T)$ in Fig. 1(b). Instead of following the trend from high temperatures, the conductivity deviates from the linear T dependence toward the $T=0$ ground state configuration. It has long been suggested that the apparent MIT observed here might be of a density inhomogeneity effect, and belongs to a general class of 2D percolation problem.[16,17] In this picture, at very low densities, the 2DES is macroscopically inhomogeneous and first forms isolated puddles by occupying the low potential "valleys." As n increases, the area of the electron puddles increases and at the percolation threshold density n_p, some puddles are connected, giving rise to a conducting path throughout the sample. Experimentally, this percolation-type 2D MIT is supported by various measurements, e.g., scanning near-field optical microscopy,[18] scanning single-electron transistor microscopy,[19] transport,[20] and surface-acoustic wave[21] experiments.

According to the percolation model, the conductivity of the 2DES follows the scaling function $\sigma \sim (n/n_p - 1)^{\alpha}$, and the exponent α in a classical percolation transition is 4/3 (Ref. 14). In Fig. 3, we show the measured σ at $T=0.3$ K as a function of $n/n_p - 1$ in a log-log scale. Since n_p is not known beforehand from finite temperature measurements, three trial densities of n_p are used, in units of 10^{11} cm^{-2}, $n_1=0.23$ (extrapolation to $\sigma_0=0$ from the low-n regime), $n_2=0.28$ (extrapolation to $\sigma_0=0$ from the high-n regime), and $n_c=0.32$(critical density of the observed 2D MIT). For $n_p=n_2=0.28\times 10^{11}$ cm^{-2}, except for some deviation at very low densities, a good linear fit with $\alpha=1.31$, close to 4/3 in the classical model, can be obtained over two decades in σ. Here, we want to point out that the deviation from the above percolation fitting occurs at the density where the "kink" in Fig. 2 is observed. This, again, suggests that the electronic phase in the low-density regime is different from that in the high-density regime. As for $n_p=n_2$, it is not known, at this stage, whether this coincidence is accidental or the two densities are actually deeply related. On the other hand, for $n_p=n_1$ or n_c, the fittings to a power-law behavior are poor. We nevertheless emphasize two important points when applying the percolation model to our data. First, the power law of conductivity in the percolation model only holds for densities very close to n_p, or $n/n_p - 1 \ll 1$.[13] Second, percolation transition is essentially a zero-temperature phase transition. As seen in Fig. 1(b), $\sigma(T)$ does not saturate at our lowest measured $T=0.3$ K. In this regard, it is necessary that further measurements be carried out at lower temperatures.

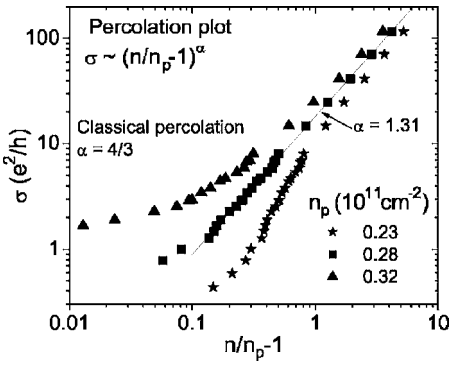

FIG. 3. Percolation plot of $\sigma(0.3\ \mathrm{K})$ vs $n/n_p - 1$ in a log-log scale. Three trial densities at 0.23, 0.28, and 0.32×10^{11} cm^{-2}, indicated by different symbols, are used as n_p in the plot. For $n_p=0.28\times 10^{11}$ cm^{-2}, excluding the two lowest density points, a linear fit (solid line) is obtained over two decades in σ. And the slope corresponds to an exponent of $\alpha=1.31$.

In summary, in a high-quality Si quantum well specimen, near the apparent 2D metal-to-insulator transition, a linear temperature dependence of conductivity is observed at T around T_F on both sides of n_c. When σ_0, the extrapolation of this linear $\sigma(T)$ to $T=0$, is plotted as a function of density, two regimes with different σ_0 vs n relations are readily seen. Interestingly, the two linear $\sigma_0(n)$ regions cross almost exactly at n_c, and σ_0 at the crossing point is e^2/h, the quantum conductance, per square. We also show that the measured $\sigma(T)$ at our low-T limit can be fitted by a percolation scaling function $\sigma \sim (n/n_p - 1)^{\alpha}$ when $n_p=n_2$, the extrapolation of $\sigma_0(n)$to $T=0$ on the high-density side.

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